

## Problem Set 2 (*Instructor: T. Pulliam*)

### Problem Set on Taylor Tables and Truncatin Error

1. Using Taylor Tables find  $er_t$  for

(a)  $(\delta_x u)_j = (u_{j+1} - u_{j-1}) / (2\Delta x)$

(b)  $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2}) / (12\Delta x)$

(c)  $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1}) / (2\Delta x)$

2. The finite difference scheme for the 1<sup>st</sup> derivative is given by

$$(\delta_x u)_j = \frac{1}{\alpha \Delta x} (-u_{j+2} + \beta u_{j+1} - \beta u_{j-1} + u_{j-2})$$

- (a) Using Taylor Tables, find the values of  $\alpha$  and  $\beta$  which result in a 4<sup>th</sup> order accurate method. *Hint: Multiply by  $\alpha \Delta x$  to make the algebra easier*
- (b) Find  $er_t$  for the method.
- (c) If we set  $\beta = 4$ , find  $\alpha$  and identify the order of accuracy.
3. Find, by means of a Taylor table, the values of  $a$ ,  $b$ ,  $c$ , and  $d$  that minimize the value of  $er_t$  in the expression

$$a \left( \frac{\partial u}{\partial x} \right)_{j-1} + \left( \frac{\partial u}{\partial x} \right)_j - \frac{1}{\Delta x} [bu_{j+1} + cu_j + du_{j-1}] = ? \quad (1)$$

What is the resulting finite difference scheme and what is the value of  $er_t$ ?

### Problem Set on Modified Wave Numbers

4. Find the expression for the modified wave number ( $ik^*$ ) in the following centered difference approximations to  $(\delta_x u)_j$  in terms of  $\Delta x$  and  $k$ . This is done just as done in class where we let  $u_j = e^{ikj\Delta x}$ . (Cast the results in terms of  $\sin(k\Delta x)$  and  $\cos(k\Delta x)$ ).

(a)  $(\delta_x u)_j = (u_{j+1} - u_{j-1}) / (2\Delta x)$

(b)  $(\delta_x u)_j = (-u_{j+2} + 8u_{j+1} - 8u_{j-1} + u_{j-2}) / (12\Delta x)$

(c)  $\frac{1}{6}((\delta_x u)_{j+1} + 4(\delta_x u)_j + (\delta_x u)_{j-1}) = (u_{j+1} - u_{j-1}) / (2\Delta x)$

5. Find the expression for the modified wave number in the following one sided difference approximations to  $(\delta_x u)_j$  in terms of  $\Delta x$  and  $k$ . In this case there will be real and imaginary parts to the modified wave number. (Cast the results in terms of  $\sin(k\Delta x)$  and  $\cos(k\Delta x)$ ).

(a)  $(\delta_x u)_j = (u_j - u_{j-1}) / \Delta x$

(b)  $(\delta_x u)_j = (3u_j - 4u_{j-1} + u_{j-2}) / (2\Delta x)$

(c)  $2(\delta_x u)_j + (\delta_x u)_{j-1} = (u_{j+1} + 4u_j - 5u_{j-1}) / (2\Delta x)$